Exam Seat No:_______________ **Enrollment No:__________________**

C.U.SHAH UNIVERSITY

Wadhwan City

Subject Code : 5SC02MTC5 **Summer Examination-2014** **Date: 18 /06/2014**

Subject Name:- **Algebra-I**

Branch/Semester:- M.Sc(Maths)/II **Time:02:00 To 5:00**

Examination: Regular

Instructions:-

(1) Attempt all Questions of both sections in same answer book / Supplementary
(2) Use of Programmable calculator & any other electronic instrument is prohibited.
(3) Instructions written on main answer Book are strictly to be obeyed.
(4) Draw neat diagrams & figures (If necessary) at right places
(5) Assume suitable & Perfect data if needed

SECTION-I

Q-1

a) If U is an ideal of a ring with unity and $1 \in U$ prove that $U = R$. (02)

b) Define principle ideal ring and give an example with illustration. (02)

c) Define integral domain. (01)

d) What are characteristics of the following rings ? (01)
(i) $(\mathbb{Z}_6, +_6, \cdot_6)$
(ii) $(\mathbb{Z}, +, \cdot)$

e) Define polynomial in an integral domain. (01)

Q-2

a) Let $R = \left\{\begin{pmatrix} a & b \\ -\bar{b} & \bar{a}\end{pmatrix} : a, b \in \mathbb{C}\right\}$, where $\bar{a}$ is the conjugate of the complex number a. show that R is a non-commutative ring with unity. (05)

b) For nonzero polynomials $f, g \in D[x]$, prove that $[fg] = [f] + [g]$. (05)

c) For ideals I_1 an I_2 of a ring R, prove that $I_1 \cup I_2$ is also an ideal of R if and only if $I_1 \subset I_2$ or $I_2 \subset I_1$. (04)

OR

Q-2

a) Consider the subset, $I = \left\{\begin{pmatrix} a & 0 \\ b & 0\end{pmatrix} : a, b \in \mathbb{Z}\right\}$ of the ring $R = (M_2(\mathbb{Z}), +, \cdot)$. Is I right ideal of ? Is I left ideal of ? (05)

b) Prove that, the characteristic of a ring R with unity is n iff n is the smallest positive integer with $n1 = 0$. (05)

c) Let $(R, +, \cdot)$ and $(R', \oplus, \ominus)$ be rings. If homomorphism $\phi: (R, +, \cdot) \to (R', \oplus, \ominus)$ is onto with its kernel K, prove that $R/K \cong R'$. (04)

Q-3

a) Prove that a finite integral domain is a field. (07)

b) Let D' be the set of all constant polynomials in $D[x]$, that is $D' = \{(a, 0, 0, \dots): a \in D\}$, prove that (07)
(i) D' is an integral domain.
(ii) $D \cong D'$.

OR

Q-3 a) For an integral domain D, show that the set $D[x]$ of all polynomials on D is also an integral domain under the binary operations addition and multiplication. (07)

b) Prove that every Euclidean ring is a principle ideal ring. (07)

SECTION-II

Q-4 a) If $f = (0, 1, 2, 0, 0, 0, \ldots)$, $g = (1, 0, -3, 1, 0, 0, \ldots)$ for $f, g \in \mathbb{Z}[x]$ find $f + g$ and $f - g$. (02)

b) If $f = ([3], [0], [1], [0], [0], [0] \ldots)$ and $g = ([2], [3], [4], [2], [0], [0])$, for $f, g \in \mathbb{Z}_5[x]$, find $f.g$. (02)

c) Define irreducible polynomial in $F[x]$. (01)

d) Define algebraic element. (01)

e) Define normal extension of a field. (01)

Q-5 a) If $x = (0, 1, 0, 0, \ldots) \in D[x]$, show that $x^n = (a_0, a_1, a_2, \ldots)$ for each positive integer n with $a_n = 1$ and $a_i = 0$ for each non-negative integer $i \neq n$. (05)

b) Obtain the quotient $q(x)$ and remainder $r(x)$ on dividing $f(x) = 4x^4 - 3x^2 + 2$ by $g(x) = x^3 - 2x + 1$ in $\mathbb{R}[x]$ and express it in the form of division algorithm. (05)

c) Find all zeros of $f(x) = 30x^3 + 89x^2 + 82x + 24$ in $\mathbb{Z}_7[x]$. (04)

OR

Q-5 a) For polynomials $f(x), g(x) \neq 0$ in $F[x]$, prove that there exist unique polynomials $q(x)$ and $r(x)$ such that $f(x) = q(x)g(x) + r(x)$ where either $r(x) = 0$ or $[r(x)] \leq [g(x)]$. (07)

b) State and prove unique factorization theorem for polynomials. (07)

Q-6 a) State and prove Eisenstein's criterion. (07)

b) Suppose K is a finite extension of a field F. show that $G(K, F)$ is a finite group and its order, $o\big(G(K, F)\big)$ satisfies the relation $o\big(G(K, F)\big) \leq [K : F]$. (07)

OR

Q-6 a) Let K be an extension of a field F. prove that the element $a \in K$ is algebraic over F if and only if $F(a)$ is a finite extension of F. (07)

b) Let K be a normal extension of a field F, of characteristic 0. Show that there is a one to one correspondence between the set of subfiels of K which contains F and the set of subgroups of $G(K, F)$. (07)

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